# Definitions and key facts for section 1.5

A system of linear equations is said to be **homogeneous** if it can be written in the form $A\mathbf{x} = \mathbf{0}$. Notice such a system is always solved by the **trivial solution** $\mathbf{x} = \mathbf{0}$. Any other solution that is nonzero is a **nontrivial solution**.

**Fact:** The homogeneous equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution if and only if the equation has at least one free variable.

A solution set described in terms of a linear combination of fixed vectors with weights varying over all real numbers is said to be in **parametric vector form**. (Usually, the free variables of the system being solved serve as the parameters.)

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Any solution to a nonhomogeneous equation $A\mathbf{x} = \mathbf{b}$ is called a **particular solution.** A solution to the **associated homogeneous equation** $A\mathbf{x} = \mathbf{0}$ is called a **homogeneous solution.**

**Fact: The solution set of a nonhomogenous system**
Suppose the equation $A\mathbf{x} = \mathbf{b}$ is consistent for some given $\mathbf{b}$, and let $\mathbf{p}$ be a solution. Then the solution set of $A\mathbf{x} = \mathbf{b}$ is the set of all vectors of the form $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$ , where $v_h$ is any solution of the homogeneous equation $A\mathbf{x} = \mathbf{0}$.